into account in the balance of forces. The Coriolis force is proportional in magnitude to the flow speed and directed perpendicular to the direction of the flow. It acts to the left of the flow in the southern hemisphere, and to the right in the northern hemisphere. A somewhat inaccurate but helpful way to see why the direction is different in the two hemispheres is related to the principle of conservation of angular momentum.

A water particle at rest at the equator carries angular momentum from the earth's rotation. When it is moved poleward it retains its angular momentum while its distance from the earth's axis is reduced. To conserve angular momentum it has to increase its rotation around the axis, just as ballet dancers increase their rate of rotation when pulling their arms towards their bodies (bringing them closer to their axis of rotation). The particle therefore starts spinning faster than the earth below it, i.e. it starts moving eastward. This results in a deflection from a straight path towards right in the northern hemisphere and towards left in the southern hemisphere. Likewise, a particle moving toward the equator from higher latitudes increases its distance from the axis of rotation and falls back in the rotation relative to the earth underneath; it starts moving westward, or again to the right in the northern and to the left in the southern hemisphere.

More on the Coriolis force can be found in Pond and Pickard (1983) or other text books. Neumann and Pierson (1966) give a detailed derivation based on Newton's Law of Motion.

The balance between the Coriolis force and the pressure gradient force is called *geostrophic balance*, and the corresponding flow is known as geostrophic flow. Compared to movement on a non-rotating earth, where the flow crosses isobars from high to low pressure, geostrophic flow is characterized by movement *along* isobars. We can see an example of geostrophic motion in the atmosphere if we recall the relation between air pressure (Figure 1.3) and wind (Figure 1.2). As already noted in Chapter 1, the wind direction nearly coincides with the orientation of the isobars. In the upper atmosphere - the analogy to the ocean interior - winds are strictly geostrophic. Winds at sea level are affected by bottom friction and therefore blow at a small angle to the isobars.

A useful quantity for the description of the oceanic circulation is mass transport. The basic definition is

$$\boldsymbol{M}^* = \rho\, \boldsymbol{v}\ . \qquad (3.1)$$

Here, $\boldsymbol{M}^*$ is the mass transport through an area of unit width (1 m^2) perpendicular to the direction of the flow and $\boldsymbol{v}$ the velocity vector with components (u,v,w) along the (x,y,z) axes. $\boldsymbol{M}^*$ is therefore a vector which points in the same direction as velocity and has units of mass per unit area and unit time, or kg m^{-2} s^{-1}. More commonly, mass transport refers to the total transport of mass in a current, *i.e.* integrated over the width and the depth of the current. It then has dimensions of mass per unit time, or kg s^{-1}. It is also possible to define the mass transport in a layer of water between depths z_1 and z_2:

$$\boldsymbol{M} = \int_{z_1}^{z_2} \rho \boldsymbol{v}\, dz \quad . \qquad (3.2)$$

Thus, $\boldsymbol{M}$ represents the transport between depths z_1 and z_2 per unit width (1 m) perpendicular to the flow and has units of mass per unit width and unit time (kg m^{-1} s^{-1}).

layer is considered motionless on account of its large vertical extent. The thickness of the upper layer or interface z = *H* (x,y,t) is allowed to vary. In the real ocean a fairly sharp density interface exists outside the polar regions, characterized by a rapid temperature change from near 20°C to below 10°C (the permanent thermocline or pycnocline, see Figure 6.1). The $1^1/_2$ layer ocean is a somewhat crude approximation of that situation but can describe flow above the pycnocline quite well.

The driving force for geostrophic currents are the horizontal differences in pressure or steric height, so we can add any arbitrary constant to the steric height field without affecting the currents deduced from it. In our $1^1/_2$ layer model, we choose a depth of no motion z_{nm} at an arbitrary depth that lies entirely in the lower layer (see Figure 3.3) and evaluate steric height $h(x,y)$ by integrating from z_{nm} upwards. Being the distance between isobaric surfaces, $h(x,y)$ is then independent of x and y in the lower layer. We conclude from eqn (3.4) that there is no geostrophic flow in the lower layer; this is consistent with the idea that the lower layer is at rest.

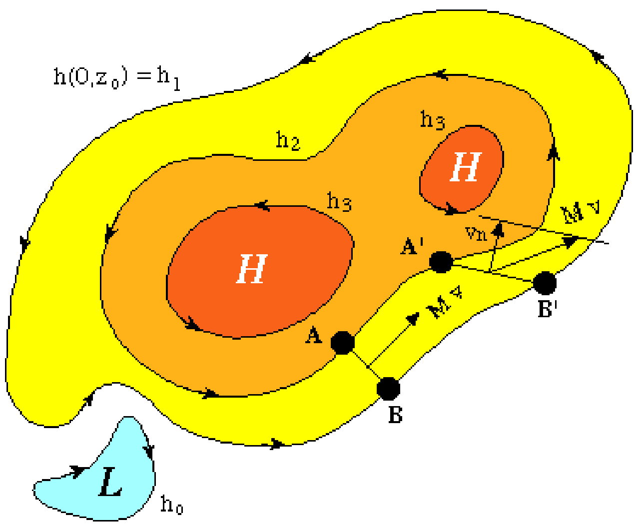

Fig. 3.2: Illustration of the relationship between a map of steric height (dynamic topography), geostrophic flow, and the evaluation of the geostrophic mass transport per unit depth *M'* between two streamlines (contours of constant steric height). For both station pairs, *A* and *B* and *A'* and *B'*, Δh in eqn (3.4) is given by $h_2 - h_1$. The geostrophic velocity is inversely proportional to the distance between streamlines, or equal to *M'* divided by density and by the distance between points *A* and *B*, because the section *AB* is perpendicular to the streamlines. If station pair *A'* and *B'* is used for the calculation, eqn. 3.4 still produces the correct geostrophic mass transport *M'* between streamlines h_1 and h_2, but the velocity derived from *M'* and distance *A'B'* is only the velocity component v_n perpendicular to the section *A'B'*. Flow direction is shown for the southern hemisphere.

This simple rule allows a very easy check on the current direction from hydrographic measurements; readers may want to verify it on Figure3.3. While Rule 1 expresses the properties of geostrophy and is thus valid wherever geostrophy holds, Rules 1a and 2 are based on the $1^1/_2$-layer model and therefore not as widely applicable. A more complete derivation based only on the assumption of geostrophy which contains the $1^1/_2$-layer ocean as a special case but applies to the continuously stratified ocean as well leads to

> **Rule 2a**: If in the southern (northern) hemisphere isopycnals slope upward to the right (left) across a current when looking in the direction of flow, current speed decreases with depth; if they slope downward, current speed increases with depth.

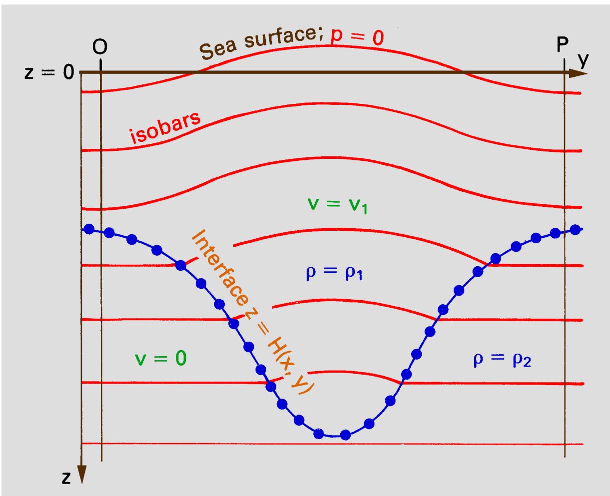

Fig. 3.3. Side view of a $1^1/_2$-layer ocean. The thermocline depth H is variable, but the density difference $\Delta\rho = \rho_2 - \rho_1$ between both layers is constant. Horizontal gradients of steric height $h(x,y)$ are independent of depth in the top layer, so by Rule 1 the currents are also independent of depth in this layer. $h(x,y)$ also measures the surface elevation, and is proportional to the thermocline depth $H(x,y)$ through eqn (3.6). The depth of no motion z_{nm} can be anywhere where it is located entirely in the lower layer.

In most oceanic situations, and in particular in the upper 1500 m of the ocean, the effect of the vertical salinity gradient on density is much smaller than the effect of the vertical temperature gradient, and the word "isopycnals" can be replaced by "isotherms". A vertical temperature section is then sufficient to get an idea of the direction of flow perpendicular to the section. In the atmosphere density is mostly determined by temperature alone, and application of Rule 2a gives a direct relationship between the structure of the temperature and the wind field. Rule 2a is therefore known as the *thermal wind relation*. The term has been adopted in oceanography; a current which is recognized by sloping isopycnals is sometimes called a "thermal wind".

For completeness we note without verification that Rule 2a is valid in western boundary currents and for zonal flow near the equator if the hydrographic section is taken perpendicular across the current axis.

$$M_{tot} = \frac{gH\ \Delta\rho\ \Delta H}{f(y_2)} = \frac{\rho_o\ gH\ \Delta h}{f(y_2)} \quad . \qquad (3.8)$$

The Coriolis parameter varies with latitude, increasing in magnitude with distance from the equator: $| f(y_1) | < | f(y_2) |$. If the depression of the interface illustrated in Figure 3.4 covers a large enough region (typically some hundreds of kilometres across), the southward transport between the two isobars is smaller between *C* and *D* than between *A* and *B*. As the water which passes between *A* and *B* has to go somewhere, some of it is pushed downwards. We conclude that on the *western* side of the eddy the interface is pushed downwards. By contrast, a similar argument shows that on the *eastern* side of the eddy flow which passes between *A*' and *B*' is larger than flow between *C*' and *D*', and the interface is pulled upwards. The net effect is a westward movement of the interface depression and with it the eddy. The same derivation of westward movement can be made for cyclonic (shallow thermocline) eddies.

The westward movement of such "planetary eddies" is known as Rossby wave propagation. Rossby waves tend to carry energy from the ocean interior into the western boundary current region of Figure 3.1. The accumulation of energy in the west leads to an intensification of the currents on the western side of all oceans; examples are the East Australian Current, the Gulf Stream, or the Agulhas Current. Our Rule 1 becomes invalid in these narrow western boundary currents, where friction and nonlinear effects lead to dissipation of energy. Because the western boundary layer is only about 100 km wide, these currents follow the coast closely and are only poorly resolved by the climatological maps of Figure 2.8, which smooth all data over horizontal distances of about 700 km. It should also be observed that much of the flow in the western boundary layers occurs over the continental slope and shelf, where steric height relative to 2000 m cannot be defined. However, the intense outflows of the western boundary currents can be seen moving eastwards from the western edge of each ocean at 30 - 40° N or S.

We can readily estimate the speed of a Rossby wave. This is done most easily and to sufficient accuracy on the so-called β-plane, which approximates the Coriolis parameter by $f = f_0 + \beta y$, i.e. a function which varies linearly with latitude. Between the two latitudes y_1 and y_2, the Coriolis parameter then changes by an amount $\beta\Delta y$, where $\Delta y = y_1 - y_2$. For small Δy, the net mass convergence between the streamlines through *A* and *B* or *C* and *D* is, from eqns (3.7) and (3.8) and noting that f is negative in the southern hemisphere,

$$g\ H\ \Delta\rho\ \Delta H \left(\frac{1}{f(y_1)} - \frac{1}{f(y_2)} \right) = g\ H\ \Delta\rho\ \Delta H \frac{\beta\ \Delta y}{f^2(y_1)} \quad , \qquad (3.9)$$

where ΔH is the difference in thermocline depth between *A* and *B* (or between *C* and *D*). This mass convergence will force the interface down over the area $\Delta x \Delta y$ defined by *A*, *B*, *C*, and *D*. For small Δx and Δy we find:

$$\rho_o \frac{\partial H}{\partial t} = \frac{gH\ \Delta\rho\ \Delta H\ \beta\ \Delta y}{f^2(y)\ \Delta x\ \Delta y} \quad , \text{or}$$

$$\frac{\partial H}{\partial t} = \frac{\beta\ g\ H}{f^2(y)} \frac{\Delta\rho}{\rho_0} \frac{\partial H}{\partial x} \quad . \qquad (3.10)$$